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A CAR-FOLLOWING MODEL

RELATING REACTION TIME AND TEMPORAL HEADWAYS

TO ACCIDENT FREQUENCY

by

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ABSTRACT:

This paper deals with a car-following model relating driver reaction time, temporal headway and deceleration response to accident frequency. The central goal is to assess the sensitivity of "collision" probability to a shift in expected reaction time. This problem eventually reduces to determining the sensitivity of the probability of "ruin" to changes in the drift of the process of cumulative reaction times. "Diffusion-type" approximations are used and it is found that additive changes in mean reaction time correspond to multiplicative changes in "collision" probability. Numerical examples are given to illustrate this profound effect.

Prepared by:

1. Introduction.

In this paper we look at a simple car-following model of traffic congestion involving driver reaction time, following distance and deceleration responses. The main result relates these parameters to the frequency (probability) of accidents. Somewhat surprisingly the conditional probability of a car being involved in an accident, given no previous accident, is shown to be a function of the cumulative effects of the reaction times and temporal headways of those cars preceding it in the traffic jam. When temporal headways are constant, determining the probability of an eventual accident amounts to determining the probability of ruin for a problem of collective risk. A shift upward (downward) of size δ in the expected reaction time causes the "ruin" barrier to increase its slope by δ. The central goal is to assess the sensitivity of this "ruin" or "collision" probability to such a shift in expected reaction time. It is shown that the change in "collision" probability corresponding to an additive shift in expected reaction time is in fact multiplicative. Thus, a shift upward in expected reaction time could explain higher accident rates in inclement weather, while a downward shift could predict the accident saving benefits of a man-machine braking system with lower reaction time. Some numerical examples are given with some empirical evidence of their validity.

2. Model Assumptions.

Assume that cars are moving along a one-way road with no passing. In effect this could apply to multilane freeway traffic during the rush hour when little passing occurs. All cars are assumed to be moving at speed v_o prior to entering a bottleneck, which causes the lead driver to slow to speed γv_o $(0 < \gamma < 1)$. We suppose that traffic is at "peak congestion" in the sense that the n^{th} car c_n has a forward spatial headway $T_n v_o$ where T_n defines c_n's minimum forward temporal headway. The length (or effective length) of the n^{th} car is denoted by Δ_n. At time $t = 0$, the lead driver c_o hits his brakes deciding to "slow down to speed γv_o, with constant deceleration rate $a_o > 0$." If $t_o \equiv$ total time for c_o to reach speed γv_o, then

$$t_o = v_o(1-\gamma)/a_o \tag{1}$$

Assume that c_1's front bumper is at $(0,0)$ in the (t,x) plane. If $X_j(t)$ represents the trajectory of c_j in the (t,x) plane, then

$$X_o(t) = \begin{cases} \Delta_o + T_1 v_o + v_o t & , \quad t \leq 0 \\ \Delta_o + T_1 v_o + v_o t - a_o t^2/2 & , \quad 0 < t \leq t_o \\ \Delta_o + T_1 v_o + v_o^2(1-\gamma^2)/2a_o + \gamma v_o(t-t_o), & \quad t \geq t_o \end{cases} \tag{2}$$

thus c_o assumes a constant speed γv_o after t_o.

It is assumed that c_j has a positive reaction time r_j during which he continues to travel at speed v_o. Thus c_j reacts at time $\sum_{i=1}^{j} r_i$. At $t = r_1$, c_1 must choose a deceleration rate a_1 just as at $t = \sum_{i=1}^{j} r_i$, c_j will choose a_j. Clearly if r_j is too large, c_j may not be able to choose a_j within the physical capabilities of himself and the car in order to avoid a collision. We will assume simply that the common upper bound on deceleration rates for all car-driver systems is A.

The critical point in building a model comes in trying to describe the decision process that a driver uses in choosing his deceleration rate. This not only varies from driver to driver, but varies with time for a given driver and with "driving conditions." Indeed, if the reaction time is sufficiently large, then the front driver may already have stopped decelerating. Specifically, if V_j^* is the minimum velocity of c_j along its trajectory, we will assume that $r_{j+1} < (v_o - V_j^*)/a_j$. Thus it will be our goal to describe and model traffic jams where reaction times are "small" and to study the sensitivity of certain operating characteristics of the traffic jam to deviations of reaction times within this limited range. Assume that c_{j+1} projects that c_j will continue decelerating to a stop. Acting accordingly, the minimum deceleration rate which c_{j+1} can choose to avoid crossing the "projected path" of c_j will be shown to be

$$a_j v_o [v_o - 2a_j (r_{j+1} - T_{j+1})]^{-1}. \qquad (3)$$

The largest rate is A. Thus, if the point is ever reached where (3) > A, then c_{j+1} will be in serious danger of crashing lest c_j stops braking in the meantime.

There seem to be two operating characteristics of interest for a given model (i.e., for a given specification of each driver's decision process):

(i) $P[c_{n+1} \text{ crashes} \mid \text{past history}]$

(ii) $P[N < \infty]$, where $N = \min[n : c_n \text{ crashes}]$

For an accident free car-following model see Brill [1].

3. Some Simple Relations.

In this section, we shall derive (3) as being the minimum deceleration rate to be chosen by c_{j+1} in order to avoid a crossing with the "projected" path of c_j as discussed in section 2.

At $t = \sum_{i=1}^{j+1} r_i$, c_{j+1} perceives a "projected trajectory" $X_j'(t)$ for c_j, where

$$X_j'(t) = \begin{cases} \Delta_j + T_{j+1}v_o + v_o t, & t < 0 \\ \Delta_j + T_{j+1}v_o + v_o t - a_j t^2/2, & 0 \leq t < t_j' \equiv v_o/a_j \\ \Delta_j + T_{j+1}v_o + v_o{}^2/2a_j, & t \geq t_j' \end{cases} \qquad (4)$$

Note that we have shifted our coordinates so that at time $t = 0$, c_{j+1}'s front bumper is at $(0,0)$. The minimum deceleration rate which c_{j+1} may choose is denoted by a^*_{j+1}. Thus a^*_{j+1} must be chosen in such a way that c_{j+1}'s resultant path would take him to speed zero at $x = T_{j+1}v_o + \Delta_j + v_o{}^2/2a_j$ at a time no earlier than $t_j' \equiv v_o/a_j$ and without collision. It will be seen that such a choice of a^*_{j+1} is possible iff $r_{j+1} \leq 2T_{j+1}$. Otherwise, c_{j+1} must choose a^*_{j+1} such that his own projected path brings him to a halt short of $x = T_{j+1}v_o + \Delta_j + v_o{}^2/2a_j$. The conditions which a^*_{j+1} must satisfy are:

(i) $T_{j+1}v_o + v_o{}^2/2a_j - r_{j+1}v_o = v_o t'_{j+1} - a^*_{j+1}t'^2_{j+1}/2$

(ii) $v_o - a^*_{j+1}t'_{j+1} = 0$

(iii) $r_{j+1} + t'_{j+1} \geq t_j'$

Equations (i) and (ii) simultaneously yield

$$a^*_{j+1} = a_j v_o [v_o - 2a_j (r_{j+1} - T_{j+1})]^{-1}. \tag{5}$$

It can easily be verified that (iii) is a sufficient condition for the two projected paths not to cross, and that (iii) is true iff $r_{j+1} \leq 2T_{j+1}$. It may also be shown that for $r_{j+1} > 2T_{j+1}$,

$$a^*_{j+1} = 2T_{j+1} v_o a_j [2T_{j+1} v_o - r^2_{j+1} a_j]^{-1}.$$

However, we shall stick to the case $r_{j+1} \leq 2T_{j+1}$.

4. Operating Characteristics.

We will assume that c_j chooses $a_j = a_j^*$ as in section 3, assuming $r_{j+1} \leq 2T_{j+1}$ and $r_{j+1} < (v_o - V_j^*)/a_j$. Thus,

$$a_{j+1} = a_j v_o [v_o - 2a_j(r_{j+1} - T_{j+1})]^{-1}, \quad j = 0,1,2,\ldots \qquad (6)$$

It follows that

$$a_j = a_o v_o [v_o - 2a_o S_j]^{-1}, \quad j = 1,2,\ldots \qquad (7)$$

where $S_j = \sum_{i=1}^{j} (r_i - T_i)$.

$$\text{Let} \quad N \equiv \begin{cases} \min\{n: a_n > A\} = \min\{n: S_n > v_o(A-a_o)/2a_oA\} \\ \infty \quad , \text{ if } S_j \leq v_o(A-a_o)/2a_oA \text{ for all } j \end{cases}$$

Clearly N is the random index of the first car that crashes. What is interesting is the cumulative effect of S_n on the probability that c_{n+1} crashes. Namely,

$$P[c_{n+1} \text{ crashes} \mid N > n, S_n] = P[r_{n+1} - T_{n+1} > v_o(A-a_o)/2a_oA - S_n]. \qquad (8)$$

Assuming that $\{r_j - T_j, j = 1,2,\ldots\}$ is an i.i.d. sequence with c.d.f. F, we recognize N as a first entrance time of the random walk $\{S_n\}$ into the set $(v_o(A-a_o)/2a_oA, \infty)$. Thus $P[N < \infty]$, the probability of an eventual crash (due to that bottleneck), is unity iff $E(r_j - T_j) \equiv \mu \geq 0$. More likely, $\mu < 0$ in which case $P[N < \infty] < 1$. It may be shown using the martingale $\{e^{\beta S_n}\}$ and

the defective stopping time N that

$$P[N < \infty] \leq \exp\{-\beta v_o (A-a_o)/2a_o A\} \tag{9}$$

where

$$\phi(\beta) = \int_0^\infty e^{\beta y}\, dF(y) = 1 \qquad (\beta>0) \tag{10}$$

Such a β is known to exist provided $F(\epsilon) < 1$ for some $\epsilon > 0$ and $\mu < 0$. See Wald [3]. The bound in (9) is a good approximation provided the excess over the boundary $S_N - v_o(A-a_o)/2a_o A$ is small. We shall use (10) in the next section to provide some numerical examples.

An important question is "How is accident frequency affected by a shift in expected driver reaction time?" This and other interesting characteristics of the model may be seen more readily if we assume $T_j \equiv T$, a constant, for all j. The problem of determining $P[N < \infty]$ then becomes one of collective risk. See Feller [2].

$$\text{i.e.,} \quad N = \min\{n: \sum_{i=1}^{n} r_i > nT + v_o(A-a_o)/2a_o A\}$$

See Figure 1.

A negative (positive) shift δ in the mean of r would increase (decrease) the slope of the barrier by δ.

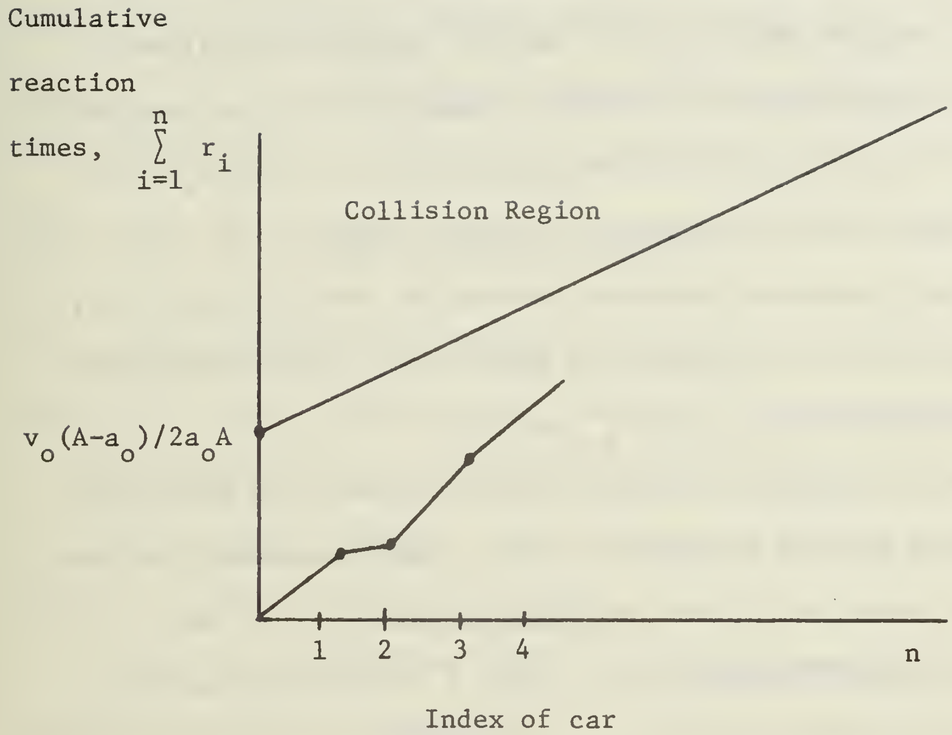

Figure 1.

For $T_j \equiv T$, a collision occurs when the random walk $\sum_{i=1}^{n} r_i$ crosses the barrier bounded by the line $nT + v_o(A-a_o)/2a_oA$.

5. A Numerical Example.

In this section we seek to determine the sensitivity of $P[N < \infty]$, the probability of eventual collision, to a shift in expected reaction time. We will use the approximation given by (9). The probability of "collision" depends on β via the distribution of $(r-T)$ as well as determination of A and a_o, so that a simple general analytical treatment seems out of reach. We will content ourselves then with fitting the distribution of r, and using this fit to determine a value for β with "reasonable" values assigned to A, a_o, v_o and T.

For tractability one need look no further than the normal distribution for the fitting of (r,T), notwithstanding the fact that this assumption allows for negative values of r and T. A sufficiently small variance on r and T will counteract this drawback.

We are assuming then that $r - T$ is distributed normal $N[\mu_r - \mu_T, \sigma^2_{r-T}]$, where $\sigma^2_{r-T} = \mathrm{Var}(r-T) = \sigma^2_r + \sigma^2_T - 2\,\mathrm{Cov}(r,T)$. Equation (10) yields

$$\exp[\sigma^2_{r-T}\beta^2/2 + \beta(\mu_r-\mu_T)] = 1$$

which has a positive root

$$\beta = 2(\mu_T-\mu_r)/\sigma^2_{r-T} \tag{11}$$

iff $\mu_r < \mu_T$.

From (9) and (11) it follows that

$$P[N < \infty] \approx \exp[-v_o(A-a_o)(\mu_T-\mu_r)/a_oA\sigma^2_{r-T}] \tag{12}$$

From (12) it may be seen immediately that a change of size $\Delta\mu_{r-T} > 0$ in μ_{r-T} would result in a multiplicative change in "collision" probability with the multiplicative factor given by

$$\exp[\Delta\mu_{r-T}v_o(A-a_o)/Aa_o\sigma^2_{r-T}] \tag{13}$$

Let us propose some "reasonable" values for the above parameters and see what happens to the "collision" probability when μ_r is altered. Let $v_o = 50$ ft/sec, $A = 20$ ft/sec^2, $a_o = 15$ ft/sec^2, $\mu_r = .45$ sec, $\mu_T = .65$ sec, $\sigma_r = .15$ sec, $\sigma_T = .15$ sec and $\mathrm{Cov}(r,T) = .0025$. Thus, $\sigma^2_{r-T} = .04$.

From (12) we find that $P[N < \infty]$ is approximately $\exp[-50/12] \approx .0155$. Assume that μ_{r-T} is increased from $-.20$ sec to $-.10$ sec, the change being due to some factor such as inclement weather. The multiplicative factor (13) may be evaluated as $\exp[25/12] \approx 8.03$ whence the new probability of "collision" is $(8.03)(.0155) \approx .125$. So a change in mean reaction time of .1 sec has increased the chances of a collision from about 1/65 to 1/8.

Actually, the multiplicative factor (13) really depends on $\Delta\mu_{r-T}/\sigma^2_{r-T}$ all other things being equal. Thus we may decrease the probability of "collision" by decreasing σ^2_{r-T} in (12), all other variables being equal.

6. Conclusions.

The above results hold of course for a very specific accident-producing situation; namely, the rear-end collision. It does not purport to explain accident frequency for the entire spectrum of accident situations or causes. It is not suggested then that a braking device with .1 second faster reaction will result in reducing total accidents to anything resembling the ratio 8/65 given in the numerical example. This cannot be so, because there are too many other causes of accidents. However, it is suggested that such a braking device may significantly cut down the rear-end type of collision.

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13. ABSTRACT

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